Gradient of tangent to circle

Hello! This lesson will discuss the equations of a tangent to a circle at a given point.Equation of a TangentLet’s consider the standard circle first: x2 + y2 = a2. We need to find the equation of the tangent to this circle at a given point, say (x1, y1). Have a look at the following figure to understand what I’m talking about.Let’s find it’s equation now.We know that the line passes through the point (x1, y1). Now if only we could find it’s slope…Turns out we can! We’ll use the fact that the line joining the center of the circle to the point of contact is perpendicular to the tangent.The slope of OP will be y1/x1. Using what we learnt in straight lines, the slope of the tangent will be –x1/y1.And that’s it! The required equation, using the two point form, will be (y – y1) = –(x1/y1)(x – x1), which, on rearrangement of terms, looks like this: xx1 + yy1 – x12 – y12 = 0.We can make it look a bit nicer. Since the point (x1, y1) lies on the circle, it’s coordinates must satisfy the circle’s equation, or x12 + y12 must equal a2.The equation of the tangent will therefore look like xx1 + yy1 – a2 = 0. Neat!And what if the circle is not standard? The method remains exactly the same – except that slope of the tangent will now be –(x1 + g)/(y1 + f). A figure for your reference:Using the two point form again, doing the rearrangements and some black magic, we’ll get the final equation as xx1 + yy1 + g(x + x1) + f(y + y1) + c = 0.And that’ll be all for this lesson.Lesson SummaryThe equation of the tangent to the circle x2 + y2 = a2 at the point (x1, y1) is xx1 + yy1 = a2The equation of the tangent to the circle x2 + y2 + 2gx + 2fy + c = 0 at the point (x1, y1) is xx1 + yy1 + g(x + x1) + f(y + y1) + c = 0.The next lesson will cover a few related examples. Level 8-9 Circle graphs for a circle, have the general equation \textcolor{red}{x}^2 + \textcolor{limegreen}{y}^2 = \textcolor{blue}{r}^2 e.g: \textcolor{red}{x}^2 + \textcolor{limegreen}{y}^2 = \textcolor{blue}{4}^2 which is the same as \textcolor{red}{x}^2 + \textcolor{limegreen}{y}^2 = \textcolor{blue}{16} Remember: r is the radius of the circle formed around centre (0,0) This is the most typical exam question you will face, if you learn the steps it really isn’t as tricky as it seems. Example: Find the equation of the tangent to the circle defined by x^2 + y^2 = \textcolor{red}{25} at the point (3, 4), shown on the axes below. Step 1: Find the gradient of the radius. Firstly, we can recognise that because \textcolor{red}{5^2} = \textcolor{red}{25}, the radius of this circle is \textcolor{red}{5}. We need to find the gradient of the radius which goes from the centre of the circle to the point (3,4). The tangent is perpendicular to the radius at that point (one of our circle theorems), meaning you can obtain the gradient of the perpendicular by taking the negative reciprocal of it. So, the gradient of the line that goes from the origin to (3,4) is \begin{aligned} \text{Gradient } &= \dfrac{\text{change in } y}{\text{change in }x} \\ &= \dfrac{4 - 0}{3 - 0} \\ &= \dfrac{4}{3} \end{aligned} Step 2: Find the gradient of the Tangent. Taking the negative reciprocal of this, we get \text{Gradient of tangent } = \textcolor{blue}{-\dfrac{3}{4}} Step 3: Complete the rest of the equation. Now we know the gradient, our straight-line equation must be y = \textcolor{blue}{-\frac{3}{4}}x + c, where c is the y-intercept that we are yet to determine. We know that this tangent passes through the point (3,4), so we can substitute these values of x and y into our straight-line equation and rearrange to find c. We get: 4 = \textcolor{blue}{-\dfrac{3}{4}} \times 3 + c 4 = -\dfrac{9}{4} + c \textcolor{limegreen}{c} = 4 + \dfrac{9}{4} = \textcolor{limegreen}{\dfrac{25}{4}} Now we’ve found c, we can express our equation of our tangent fully: y = \textcolor{blue}{-\dfrac{3}{4}}x + \textcolor{limegreen}{\dfrac{25}{4}} We can see that the radius of this circle extends a distance of 10 away from the centre at (0,0). Therefore, because 10^2 = 100, the equation of the circle is x^2 + y^2 = 100 First, we need to find the gradient of the line from the centre to (12, 5). \text{Gradient of radius } = \dfrac{\text{change in } y}{\text{change in }x} = \dfrac{5 - 0}{12 - 0} = \dfrac{5}{12} Now, by observing that this line is a radius, and that tangents are perpendicular to the radius, we can find the gradient of the tangent by taking the negative reciprocal of the answer we got above. \text{Gradient of tangent } = -\dfrac{12}{5} So, we know the straight-line equation for our tangent must be of the form y = -\dfrac{12}{5}x + c, where c is the y-intercept which we must determine. To do this, we can substitute the values of x = 12 and y = 5 into the straight-line equation, since we know the line must pass through those coordinates. We get the following. 5 = -\dfrac{12}{5} \times 12 + c 5 = -\dfrac{144}{5} + c c = 5 + \dfrac{144}{5} = \dfrac{169}{5} So, our final answer is y = -\dfrac{12}{5}x + \dfrac{169}{5} First, we need to find the gradient of the line from the centre to (-8, -7). \text{Gradient of radius } = \dfrac{\text{change in } y}{\text{change in }x} = \dfrac{-7 - 0}{-8 - 0} = \dfrac{7}{8} Now, by observing that this line is a radius, and that tangents are perpendicular to the radius, we can find the gradient of the tangent by taking the negative reciprocal of the answer we got above. \text{Gradient of tangent } = -\dfrac{8}{7} So, we know the straight-line equation for our tangent must be of the form y = -\dfrac{8}{7}x + c, where c is the y-intercept which we must determine. To do this, we can substitute the values of x = -8 and y = -7 into the straight-line equation, since we know the line must pass through those coordinates. We get the following. -7 =-\dfrac{8}{7} \times -8 + c -7 = \dfrac{64}{7} + c c=-\dfrac{113}{7} So, our final answer is y = -\dfrac{8}{7}x -\dfrac{113}{7} Rearrange the equation for the straight line to the form: ax+b=y and substitute this result for y into the equation for the circle (x-c)^2+(y-d)^2=e and you can rearrange to get a quadratic in x. Solve the quadratic. Since the line is a tangent to the circle, there will only be one repeated solution, which gives the x coordinate you are after. Substitute back into the equation of the line to find your y value.Ask educator a questionThe equation of the circle is: (x-a)^2 + (y-b)^2 = r^2 The equation of the tangent (linear ofc) is: y=mx+c At the point of contact for these lines, these functions will share the same co-ordinate x & y values. Therefore, you can substitute the value of the tangent (mx+c) into the value of y in the circle equation. This gives: (x-a)^2 + ((mx+c)-b)^2 = r^2 If you simplify the above, you can find 2 values for x. You can plug both values of x into the equation for the tangent to produce two y values. You now have 2 pairs of (x,y) co-ordinates. From a graphical sketch of the circle and the tangent, you can deduce which of the two co-ordinates is valid.Ask educator a questionHi A point of contact between a tangent and a circle is the only point touching the circle by this line, The point can be found either by : equating the equations; The line : y = mx +c The circle : (x-a)^2 + (y_b)^2 = r^2 The result will be the value of {x}which can be substituted in the equation of the line to find the value of the {y}. Or; by substituting the equation of the line in the equation of the circle to find the value of the {x} and then substitute this value again in the equation of the line to find the value of the {y}. Hope this will help.Ask educator a questionIt really depends on the information you are given. If you have the equations of both lines, you can do some algebra. Equations of circles are in the form (x-a)^2 + (y-b)^2 = r^2 And straight lines are y=mx+c With these you can just substitute the second equation into the first by replacing all the y's with mx+c. After you simplify, you should get a quadratic expression and solving that will give you an x coordinate. Using that you plug it back into y=mx+c to get the y coordinate. There are other ways to do this which require different peices of information. If you tell me what is given in the question, then I can tell you a more suitable methodAsk educator a questionLet's answer each one of these parts in turn: Firstly how to find the tangent of a curve. Most of the curves you will see through GCSE and A-Level are polynomials. To get a tangent line in the standard form of y=mx+c (where m is the gradient and c is the y-intercept) we first need to find the gradient. We can either differentiate the curve (very basically you time the coefficient of each term with the power of x and take one off the power) and the substitute the tangent point into the equation or you can draw a tangent line from the point and calculate the gradient using 2 points on the line. The gradient is the difference in the y co-ordinates/ difference in the x co-ordinates. Then to calculate the y-intercept we can substitute the tangent point in the equation y=mx+c with your recently calculated gradient m. Now calculating tangents to circle. The equation for a tangent to a circle is y-b=m(x-a). We can use the methods above to calculate the gradient and (a,b) is the centre of the circle. I hope this helps.Ask educator a questionmore specifically than above, but following through with the algebra: if the circle has an equation in the format: ((x-a)^2 + (y-b)^2 = r^2 .... (1) [circle centred at coordinates (a,b) with radius r] and the tangent (or line) has equation in the format: y = mx + c ...... (2) [line with slope m, intercept c] can substitute the second equation into the first (where y appears in first just use mx + c) to have a single quadratic equation in terms of x (after expanding and combining x^2, x & “non-x” terms). lastly use quadratic formulae to solve for 2 potential solutions & substitute each solution back into the line equation (y = mx + c) to solve for the corresponding y coordinate. solution for x generically is as follows: (m^2 + 1)x^2 - 2(a + b - c)x + (a^2 + (b-c)^2) - r^2 = 0 follow instructions above + use quadratic formulae to complete.Ask educator a questionthe point of contact = points of intersections to find this you can equate the both equations of the circle and the tangent then rearrange to find the x value and substitute to find y values you can do this eg/ by simultaneous equations. Ask educator a questionFind the gradient of the line from the centre of the circle to the point on the circle you need, then use that to find the gradient of the tangent. Finally, use y=mx+c with this gradient and the point on the circle it touches. Ask educator a questionWhat is 99999 divided bÿ482936How to find volume with area and density?Get an answer in 5 minutesWe'll notify as soon as your question has been answered.Ask a question to our educators50% discount available Find the equation of the circle through the points \(A(2,0)\), \(B(8,0)\), and \(C(10,4)\)... Given any three points that do not lie on a straight line, there is a unique circle passing through these three points, whose centre is the circumcentre of the triangle. The circumcentre can be found at the intersection of the perpendicular bisectors of the three sides. The circumcentre of \(ABC\) Our three points are \(A=(2,0)\), \(B=(8,0)\), and \(C=(10,4)\). The line segment between \((2,0)\) and \((8,0)\) is part of the \(x\)-axis, with midpoint \(\left(\dfrac{2+8}{2},\dfrac{0+0}{2}\right)=(5,0)\). So the perpendicular bisector of the side \(AB\) is the line \(x=5\). The line segment between \((8,0)\) and \((10,4)\) has gradient \(\dfrac{4}{2}=2\) and midpoint \(\left(\dfrac{8+10}{2},\dfrac{0+4}{2}\right)=(9,2)\). So the perpendicular bisector of the side \(BC\) is the line with gradient \(-\dfrac{1}{2}\) and passing through the point \((9,2)\). Substituting this information into the general equation for a line \(y=mx+c\), we find that the perpendicular bisector of the side \(BC\) is the line \(y=-\dfrac{1}{2}x+\dfrac{13}{2}\). The perpendicular bisector of side \(CA\), which turns out to be the line \(y=-2x+14\), also goes through the circumcentre. Substituting \(x=5\) into \(y=-\dfrac{1}{2}x + \dfrac{13}{2}\), we get \(y=4\). So these two perpendicular bisectors intersect at the point \((5,4)\), making this the circumcentre. Now we can find the radius of the circle—simply find the distance between the centre and one of the points given. The distance between \(A=(2,0)\) and \((5,4)\) is \(\sqrt{(2-5)^2+(0-4)^2}=\sqrt{25}=5\). So the circle passing through these three points has centre \((5,4)\) and radius \(5\). This means that it has equation \[(x-5)^2 + (y-4)^2 = 25.\] An alternative method might argue that we know that \(P\), the centre of the circle, lies on \(x = 5\), so let’s say \(P\) is \((5,p)\). We also know that \(PB=PC\); this equation enables us to find \(p\), and the rest follows. ... and prove that it touches the \(y\)-axis. The centre is at (5,4) and the radius is 5, so the circle must touch the y-axis. Without the use of tables [or calculators] or measurement, find the equation of the other tangent to this circle from the origin. Any line passing through the origin that is not the \(y\)-axis must have equation \(y=mx\) for some real number \(m\). We want to find an \(m\) such that \(y=mx\) and \((x-5)^2+(y-4)^2=25\) intersect at exactly one point. Substituting \(y=mx\) into \((x-5)^2+(y-4)^2=25\) and expanding, we get a quadratic equation in \(x\): \[\begin{align*} &(x-5)^2 + (mx-4)^2 = 25\\ \iff\quad& x^2 - 10x + 25 + m^2x^2 - 8mx + 16 = 25\\ \iff\quad& (m^2 +1)x^2 + (-10 - 8m)x+16 = 0. \end{align*}\] For \(y=mx\) to be a tangent to the circle, this quadratic equation must have only one root, that is, its discriminant must be zero. The discriminant of the quadratic equation \(ax^2+bx+c = 0\) is \(b^2-4ac\). The equation has a repeated root if, and only if, the discriminant is zero. Now \[\begin{align*} &(-10-8m)^2 - 4 \times (m^2+1) \times 16 = 0\\ \iff\quad& 100+160m+64m^2-64(m^2+1)=0\\ \iff\quad& 100+160m+64m^2-64m^2-64=0\\ \iff\quad& 160m+36=0\\ \iff\quad& m=-\frac{9}{40}. \end{align*}\] So \(y=-\dfrac{9}{40}x\) is the equation of this tangent. Note that in general we would expect to have two solutions for \(m\) from \(b^2=4ac\). When the origin \(O\) is outside the circle, there are exactly two tangents to the circle from \(O\). Here, however, one of our tangents is the \(y\)-axis, that has infinite gradient. We could see the equation \(100+160m+64m^2-64m^2-64=0\) as having one solution for \(m\) that is infinite. The equation of a circle can be found using the centre and radius. The discriminant can determine the nature of intersections between two circles or a circle and a line to prove for tangency.