I'm not a robot

La historia de las matemáticas es un tema fascinante que abarca miles de años de desarrollo y evolución de la ciencia matemática. Desde la antigua Mesopotamia hasta las teorías modernas de la relatividad y la mecánica cuántica, las matemáticas han sido un elemento crucial en el desarrollo de nuestra comprensión del mundo y el universo que nos rodea.Una forma popular de explorar la historia de las matemáticas es a través de una línea de tiempo. Una línea de tiempo de la historia de las matemáticas es una herramienta visual que muestra las principales contribuciones y avances en el campo de las matemáticas a lo largo del tiempo. Desde la invención del cero hasta la teoría de los números complejos, una línea de tiempo histórica de las matemáticas es una forma efectiva de aprender y entender el desarrollo de esta ciencia fundamental. En esta presentación, exploraremos algunos de los momentos más importantes en la historia de las matemáticas y veremos cómo cada uno de ellos ha influido en nuestro conocimiento y comprensión del mundo que nos rodea.Índice Las matemáticas son una disciplina que ha evolucionado a lo largo de miles de años, desde sus primeros conceptos primitivos hasta las complejas teorías y aplicaciones que conocemos hoy en día. En esta línea del tiempo se presentará de manera resumida la evolución de las matemáticas a lo largo de la historia.AntigüedadLas primeras formas de matemáticas se desarrollaron en la antigua Mesopotamia y Egipto, donde se utilizaban para medir terrenos, construir edificios y llevar registros de comercio. En Babilonia se desarrolló el sistema sexagesimal, que utiliza la base 60 en lugar de la base 10 utilizada en la actualidad. Los egipcios también desarrollaron técnicas para resolver ecuaciones lineales y cuadráticas.Grecia antiguaLos griegos fueron los primeros en desarrollar una teoría matemática formal. Pitágoras fue uno de los primeros en establecer la relación entre los números y la geometría. Euclides, en su obra "Elementos", presentó los principios básicos de la geometría y estableció los axiomas que se utilizan en la actualidad. También se desarrollaron conceptos como la trigonometría, la teoría de los números y la geometría analítica.Edad MediaEn la Edad Media se produjo un gran avance en la matemática islámica. Al-Khwarizmi, considerado el padre del álgebra, desarrolló técnicas para resolver ecuaciones lineales y cuadráticas. También se desarrollaron conceptos como el cálculo de funciones trigonométricas y la geometría esférica.RenacimientoEn el Renacimiento se produjo un gran avance en la matemática europea. Se desarrollaron conceptos como el cálculo diferencial e integral, la geometría analítica y la estadística. Figuras como Galileo Galilei, Johannes Kepler y René Descartes contribuyeron al desarrollo de la matemática moderna.Siglo XVIII y XIXEn el siglo XVIII se produjo un gran avance en el cálculo, con la aparición de figuras como Leonhard Euler, quien desarrolló la notación matemática moderna y contribuyó a la teoría de los números y la geometría. En el siglo XIX se produjo el desarrollo del álgebra abstracta y la teoría de grupos, el análisis complejo y la geometría no euclidiana.Siglo XX y actualidadEn el siglo XX se produjo el desarrollo de la teoría de conjuntos, la topología algebraica, la teoría de categorías y la teoría de la probabilidad. También se produjo el desarrollo de la informática y la matemática computacional, que han permitido el desarrollo de aplicaciones prácticas en campos como la física, la ingeniería y la economía.Cómo Surgieron las Matemáticas: Una Historia FascinanteLas matemáticas son una disciplina que ha jugado un papel fundamental en la historia de la humanidad. Desde la antigua Mesopotamia hasta la actualidad, las matemáticas han sido una herramienta indispensable para la resolución de problemas complejos en áreas como la física, la economía y la ingeniería.La historia de las matemáticas se remonta a la antigua civilización egipcia, donde se utilizaban números para la contabilidad y el comercio. Sin embargo, fue en la antigua Mesopotamia donde se desarrolló el sistema de numeración que utilizamos hoy en día, basado en el número 10.En la antigua Grecia, los matemáticos como Pitágoras y Euclides sentaron las bases de la geometría y la aritmética modernas. Fue en esta época cuando se comenzó a utilizar el método deductivo para demostrar teoremas y proposiciones matemáticas.En la Edad Media, los matemáticos árabes hicieron importantes contribuciones al campo de las matemáticas, especialmente en el ámbito de la álgebra. Fue gracias a ellos que se introdujeron los símbolos matemáticos que utilizamos hoy en día, como el signo de la suma (+) y el signo de la multiplicación (x).En la Edad Moderna, los matemáticos europeos como Newton y Leibniz desarrollaron el cálculo diferencial e integral, una herramienta fundamental para la física y la ingeniería. Fue gracias al cálculo que se pudieron describir matemáticamente fenómenos naturales como la caída de los cuerpos y el movimiento de los planetas.Hoy en día, las matemáticas siguen siendo una disciplina en constante evolución, con nuevas teorías y métodos que se desarrollan constantemente. Desde la inteligencia artificial hasta la criptografía, las matemáticas son una herramienta fundamental en muchos ámbitos de la vida moderna.Desde la antigua Mesopotamia hasta la actualidad, las matemáticas han sido una herramienta indispensable para la resolución de problemas complejos en áreas como la física, la economía y la ingeniería. A proof from Euclid's Elements (c. 300 BC), widely considered the most influential textbook of all time.[1] Part of a series onMathematics History Index Areas Number theory Geometry Algebra Calculus and Analysis Discrete mathematics Logic Set theory Probability Statistics and Decision theory Relationship with sciences Physics Chemistry Geosciences Computation Biology Linguistics Economics Philosophy Education Mathematics Portalvte The history of mathematics deals with the origin of discoveries in mathematics and the mathematical methods and notation of the past. Before the modern age and the worldwide spread of knowledge, written examples of new mathematical developments have come to light only in a few locales. From 3000 BC the Mesopotamian states of Sumer, Akkad and Assyria, followed closely by Ancient Egypt and the Levantine state of Ebla began using arithmetic, algebra and geometry for purposes of taxation, commerce, trade and also in the field of astronomy to record time and formulate calendars. The earliest mathematical texts available are from Mesopotamia and Egypt – Plimpton 322 (Babylonian c. 2000 – 1900 BC),[2] the Rhind Mathematical Papyrus (Egyptian c. 1800 BC)[3] and the Moscow Mathematical Papyrus (Egyptian c. 1890 BC). All of these texts mention the so-called Pythagorean triples, so, by inference, the Pythagorean theorem seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry. The study of mathematics as a "demonstrative discipline" began in the 6th century BC with the Pythagoreans, who coined the term "mathematics" from the ancient Greek μάθημα (mathema), meaning "subject of instruction".[4] Greek mathematics greatly refined the methods (especially through the introduction of deductive reasoning and mathematical rigor in proofs) and expanded the subject matter of mathematics.[5] The ancient Romans used applied mathematics in surveying, structural engineering, mechanical engineering, bookkeeping, creation of lunar and solar calendars, and even arts and crafts. Chinese mathematics made early contributions, including a place value system and the first use of negative numbers.[6][7] The Hindu–Arabic numeral system and the rules for the use of its operations, in use throughout the world today evolved over the course of the first millennium AD in India and were transmitted to the Western world via Islamic mathematics through the work of Muḥammad ibn Mūsā al-Khwārizmī.[8][9] Islamic mathematics, in turn, developed and expanded the mathematics known to these civilizations.[10] Contemporaneous with but independent of these traditions were the mathematics developed by the Maya civilization of Mexico and Central America, where the concept of zero was given a standard symbol in Maya numerals. Many Greek and Arabic texts on mathematics were translated into Latin from the 12th century onward, leading to further development of mathematics in Medieval Europe. From ancient times through the Middle Ages, periods of mathematical discovery were often followed by centuries of stagnation.[11] Beginning in Renaissance Italy in the 15th century, new mathematical developments, interacting with new scientific discoveries, were made at an increasing pace that continues through the present day. This includes the groundbreaking work of both Isaac Newton and Gottfried Wilhelm Leibniz in the development of infinitesimal calculus during the course of the 17th century and following discoveries of German mathematicians like Carl Friedrich Gauss and David Hilbert. The origins of mathematical thought lie in the concepts of number, patterns in nature, magnitude, and form.[12] Modern studies of animal cognition have shown that these concepts are not unique to humans. Such concepts would have been part of everyday life in hunter-gatherer societies. The idea of the "number" concept evolving gradually over time is supported by the existence of languages which preserve the distinction between "one", "two", and "many", but not of numbers larger than two.[12] The use of yarn by Neanderthals some 40,000 years ago at a site in Abri du Maras in the south of France suggests they knew basic concepts in mathematics.[13][14] The Ishango bone, found near the headwaters of the Nile river (northeastern Congo), may be more than 20,000 years old and consists of a series of marks carved in three columns running the length of the bone. Common interpretations are that the Ishango bone shows either a tally of the earliest known demonstration of sequences of prime numbers[15][failed verification] or a six-month lunar calendar.[16] Peter Rudman argues that the development of the concept of prime numbers could only have come about after the concept of division, which he dates to after 10,000 BC, with prime numbers probably not being understood until about 500 BC. He also writes that "no attempt has been made to explain why a tally of something should exhibit multiples of two, prime numbers between 10 and 20, and some numbers that are almost multiples of 10."[17] The Ishango bone, according to scholar Alexander Marshack, may have influenced the later development of mathematics in Egypt as, like some entries on the Ishango bone, Egyptian arithmetic also made use of multiplication by 2; this however, is disputed.[18] Predynastic Egyptians of the 5th millennium BC pictorially represented geometric designs. It has been claimed that megalithic monuments in England and Scotland, dating from the 3rd millennium BC, incorporate geometric ideas such as circles, ellipses, and Pythagorean triples in their design.[19] All of the above are disputed however, and the currently oldest undisputed mathematical documents are from Babylonian and dynastic Egyptian sources.[20] Main article: Babylonian mathematics See also: Plimpton 322 Babylonian mathematics refers to any mathematics of the peoples of Mesopotamia (modern Iraq) from the days of the early Sumerians through the Hellenistic period almost to the dawn of Christianity.[21] The majority of Babylonian mathematical work comes from two widely separated periods: The first few hundred years of the second millennium BC (Old Babylonian period), and the last few centuries of the first millennium BC (Seleucid period).[22] It is named Babylonian mathematics due to the central role of Babylon as a place of study. Later under the Arab Empire, Mesopotamia, especially Baghdad, once again became an important center of study for Islamic mathematics. Geometry problem on a clay tablet belonging to a school for scribes; Susa, first half of the 2nd millennium BC In contrast to the sparsity of sources in Egyptian mathematics, knowledge of Babylonian mathematics is derived from more than 400 clay tablets unearthed since the 1850s.[23] Written in Cuneiform script, tablets were inscribed whilst the clay was moist, and baked hard in an oven or by the heat of the sun. Some of these appear to be graded homework.[24] The earliest evidence of written mathematics dates back to the ancient Sumerians, who built the earliest civilization in Mesopotamia. They developed a complex system of metrology from 3000 BC that was chiefly concerned with administrative/financial counting, such as grain allotments, workers, weights of silver, or even liquids, among other things.[25] From around 2500 BC onward, the Sumerians wrote multiplication tables on clay tablets and dealt with geometrical exercises and division problems. The earliest traces of the Babylonian numerals also date back to this period.[26] The Babylonian mathematical tablet Plimpton 322, dated to 1800 BC. Babylonian mathematics were written using a sexagesimal (base-60) numeral system.[23] From this derives the modern-day usage of 60 seconds in a minute, 60 minutes in an hour, and 360 (60 × 6) degrees in a circle, as well as the use of seconds and minutes of arc to denote fractions of a degree. It is thought the sexagesimal system was initially used by Sumerian scribes because 60 can be evenly divided by 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30,[23] and for scribes (doling out the aforementioned grain allotments, recording weights of silver, etc.) being able to easily calculate by hand was essential, and so a sexagesimal system is pragmatically easier to calculate by hand with; however, there is the possibility that using a sexagesimal system was an ethno-linguistic phenomenon (that might not ever be known), and not a mathematical/practical decision.[27] Also, unlike the Egyptians, Greeks, and Romans, the Babylonians had a place-value system, where digits written in the left column represented larger values, much as in the decimal system. The power of the Babylonian notational system lay in that it could be used to represent fractions as easily as whole numbers; thus multiplying two numbers that contained fractions was no different from multiplying integers, similar to modern notation. The notational system of the Babylonians was the best of any civilization until the Renaissance, and its power allowed it to achieve remarkable computational accuracy; for example, the Babylonian tablet YBC 7289 gives an approximation of √2 accurate to five decimal places.[28] The Babylonians lacked, however, an equivalent of the decimal point, and so the place value of a symbol often had to be inferred from the context.[22] By the Seleucid period, the Babylonians had developed a zero symbol as a placeholder for empty positions; however it was only used for intermediate positions.[22] This zero sign does not appear in terminal positions, thus the Babylonians came close but did not develop a true place value system.[22] Other topics covered by Babylonian mathematics include fractions, algebra, quadratic and cubic equations, and the calculation of regular numbers, and their reciprocal pairs.[29] The tablets also include multiplication tables and methods for solving linear, quadratic equations and cubic equations, a remarkable achievement for the time.[30] Tablets from the Old Babylonian period also contain the earliest known statement of the Pythagorean theorem.[31] However, as with Egyptian mathematics, Babylonian mathematics shows no awareness of the difference between exact and approximate solutions, or the solvability of a problem, and most importantly, no explicit statement of the need for proofs or logical principles.[24] Main article: Egyptian mathematics Image of Problem 14 from the Moscow Mathematical Papyrus. The problem includes a diagram indicating the dimensions of the truncated pyramid. Egyptian mathematics refers to mathematics written in the Egyptian language. From the Hellenistic period, Greek replaced Egyptian as the written language of Egyptian scholars. Mathematical study in Egypt later continued under the Arab Empire as part of Islamic mathematics, when Arabic became the written language of Egyptian scholars. Archaeological evidence has suggested that the Ancient Egyptian counting system had origins in Sub-Saharan Africa.[32] Also, fractal geometry designs which are widespread among Sub-Saharan African cultures are also found in Egyptian architecture and cosmological signs.[33] The most extensive Egyptian mathematical text is the Rhind papyrus (sometimes also called the Ahmes Papyrus after its author), dated to c. 1650 BC but likely a copy of an older document from the Middle Kingdom of about 2000–1800 BC.[34] It is an instruction manual for students in arithmetic and geometry. In addition to giving area formulas and methods for multiplication, division and working with unit fractions, it also contains evidence of other mathematical knowledge,[35] including composite and prime numbers; arithmetic, geometric and harmonic means; and simplistic understandings of both the Sieve of Eratosthenes and perfect number theory (namely, that of the number 6).[36] It also shows how to solve first order linear equations[37] as well as arithmetic and geometric series.[38] Another significant Egyptian mathematical text is the Moscow papyrus, also from the Middle Kingdom period, dated to c. 1890 BC.[39] It consists of what are today called word problems or story problems, which were apparently intended as entertainment. One problem is considered to be of particular importance because it gives a method for finding the volume of a frustum (truncated pyramid). Finally, the Berlin Papyrus 6619 (c. 1800 BC) shows that ancient Egyptians could solve a second-order algebraic equation.[40] Main article: Greek mathematics The Pythagorean theorem. The Pythagoreans are generally credited with the first proof of the theorem. Greek mathematics refers to the mathematics written in the Greek language from the time of Thales of Miletus (~600 BC) to the closure of the Academy of Athens in 529 AD.[41] Greek mathematicians lived in cities spread over the entire Eastern Mediterranean, from Italy to North Africa, but were united by culture and language. Greek mathematics of the period following Alexander the Great is sometimes called Hellenistic mathematics.[42] Greek mathematics was much more sophisticated than the mathematics that had been developed by earlier cultures. All surviving records of pre-Greek mathematics show the use of inductive reasoning, that is, repeated observations used to establish rules of thumb. Greek mathematicians, by contrast, used deductive reasoning. The Greeks used logic to derive conclusions from definitions and axioms, and used mathematical rigor to prove them.[43] Greek mathematics is thought to have begun with Thales of Miletus (c. 624–c.546 BC) and Pythagoras of Samos (c. 582–c. 507 BC). Although the extent of the influence is disputed, they were probably inspired by Egyptian and Babylonian mathematics. According to legend, Pythagoras traveled to Egypt to learn mathematics, geometry, and astronomy from Egyptian priests. Thales used geometry to solve problems such as calculating the height of pyramids and the distance of ships from the shore. He is credited with the first use of deductive reasoning applied to geometry, by deriving four corollaries to Thales' Theorem. As a result, he has been hailed as the first true mathematician and the first known individual to whom a mathematical discovery has been attributed.[44] Pythagoras established the Pythagorean School, whose doctrine it was that mathematics ruled the universe and whose motto was "All is number".[45] It was the Pythagoreans who coined the term "mathematics", and with whom the study of mathematics for its own sake begins. The Pythagoreans are credited with the first proof of the Pythagorean theorem,[46] though the statement of the theorem has a long history, and with the proof of the existence of irrational numbers.[47][48] Although he was preceded by the Babylonians, Indians and the Chinese,[49] the Neopythagorean mathematician Nicomachus (60–120 AD) provided one of the earliest Greco-Roman multiplication tables, whereas the oldest extant Greek multiplication table is found on a wax tablet dated to the 1st century AD (now found in the British Museum).[50] The association of the Neopythagoreans with the Western invention of the multiplication table is evident in its later Medieval name: the mensa Pythagorica.[51] Plato (428/427 BC – 348/347 BC) is important in the history of mathematics for inspiring and guiding others.[52] His Platonic Academy, in Athens, became the mathematical center of the world in the 4th century BC, and it was from this school that the leading mathematicians of the day, such as Eudoxus of Cnidus (c. 390 - c. 340 BC), came.[53] Plato also discussed the foundations of mathematics,[54] clarified some of the definitions (e.g. that of a line as "breadthless length"). Eudoxus developed the method of exhaustion, a precursor of modern integration[55] and a theory of ratios that avoided the problem of incommensurable magnitudes.[56] The former allowed the calculations of areas and volumes of curvilinear figures,[57] while the latter enabled subsequent geometers to make significant advances in geometry. Though he made no specific technical mathematical discoveries, Aristotle (384–c. 322 BC) contributed significantly to the development of mathematics by laying the foundations of logic.[58] One of the oldest surviving fragments of Euclid's Elements, found at Oxyrhynchus and dated to circa AD 100. The diagram accompanies Book II, Proposition 5.[59] In the 3rd century BC, the premier center of mathematical education and research was the Musaeum of Alexandria.[60] It was there that Euclid (c. 300 BC) taught, and wrote the Elements, widely considered the most successful and influential textbook of all time.[1] The Elements introduced mathematical rigor through the axiomatic method and is the earliest example of the format still used in mathematics today, that of definition, axiom, theorem, and proof. Although most of the contents of the Elements were already known, Euclid arranged them into a single, coherent logical framework.[61] The Elements was known to all educated people in the West up through the middle of the 20th century and its contents are still taught in geometry classes today.[62] In addition to the familiar theorems of Euclidean geometry, the Elements was meant as an introductory textbook to all mathematical subjects of the time, such as number theory, algebra and solid geometry,[61] including proofs that the square root of two is irrational and that there are infinitely many prime numbers. Euclid also wrote extensively on other subjects, such as conic sections, optics, spherical geometry, and mechanics, but only half of his writings survive.[63] Archimedes used the method of exhaustion to approximate the value of pi. Archimedes (c. 287–212 BC) of Syracuse, widely considered the greatest mathematician of antiquity,[64] used the method of exhaustion to calculate the area under the arc of a parabola with the summation of an infinite series, in a manner not too dissimilar from modern calculus.[65] He also showed one could use the method of exhaustion to calculate the value of π with as much precision as desired, and obtained the most accurate value of π then known, 3+10/71 < π < 3+10/70.[66] He also studied the spiral bearing his name, obtained formulas for the volumes of surfaces of revolution (paraboloid, ellipsoid, hyperboloid),[65] and an ingenious method of exponentiation for expressing very large numbers.[67] While he is also known for his contributions to physics and several advanced mechanical devices, Archimedes himself placed far greater value on the products of his thought and general mathematical principles.[68] He regarded as his greatest achievement his finding of the surface area and volume of a sphere, which he obtained by proving these are 2/3 the surface area and volume of a cylinder circumscribing the sphere.[69] Apollonius of Perga made significant advances in the study of conic sections. Apollonius of Perga (c. 262–190 BC) made significant advances to the study of conic sections, showing that one can obtain all three varieties of conic section by varying the angle of the plane that cuts a double-napped cone.[70] He also coined the terminology in use today for conic sections, namely parabola ("place beside" or "comparison"), "ellipse" ("deficiency"), and "hyperbola" ("a throw beyond").[71] His work Conics is one of the best known and preserved mathematical works from antiquity, and in it he derives many theorems concerning conic sections that would prove invaluable to later mathematicians and astronomers studying planetary motion, such as Isaac Newton.[72] While neither Apollonius nor any other Greek mathematicians made the leap to coordinate geometry, Apollonius' treatment of curves is in some ways similar to the modern treatment, and some of his work seems to anticipate the development of analytical geometry by Descartes some 1800 years later.[73] Around the same time, Eratosthenes of Cyrene (c. 276–194 BC) devised the Sieve of Eratosthenes for finding prime numbers.[74] The 3rd century BC is generally regarded as the "Golden Age" of Greek mathematics, with advances in pure mathematics henceforth in relative decline.[75] Nevertheless, in the centuries that followed significant advances were made in applied mathematics, most notably trigonometry, largely to address the needs of astronomers.[75] Hipparchus of Nicaea (c. 190–120 BC) is considered the founder of trigonometry for compiling the first known trigonometric table, and to him is also due the systematic use of the 360 degree circle.[76] Heron of Alexandria (c. 10–70 AD) is credited with Heron's formula for finding the area of a scalene triangle and with being the first to recognize the possibility of negative numbers possessing square roots.[77] Menelaus of Alexandria (c. 100 AD) pioneered spherical trigonometry through Menelaus' theorem.[78] The most complete and influential trigonometric work of antiquity is the Almagest of Ptolemy (c. AD 90–168), a landmark astronomical treatise whose trigonometric tables would be used by astronomers for the next thousand years.[79] Ptolemy is also credited with Ptolemy's theorem for deriving trigonometric quantities, and the most accurate value of π outside of China until the medieval period, 3.1416.[80] Title page of the 1621 edition of Diophantus' Arithmetica, translated into Latin by Claude Gaspard Bachet de Méziriac. Following a period of stagnation after Ptolemy, the period between 250 and 350 AD is sometimes referred to as the "Silver Age" of Greek mathematics.[81] During this period, Diophantus made significant advances in algebra, particularly indeterminate analysis, which is also known as "Diophantine analysis".[82] The study of Diophantine equations and Diophantine approximations is a significant area of research to this day. His main work was the Arithmetica, a collection of 150 algebraic problems dealing with exact solutions to determinate and indeterminate equations.[83] The Arithmetica had a significant influence on later mathematicians, such as Pierre de Fermat, who arrived at his famous Last Theorem after trying to generalize a problem he had read in the Arithmetica (that of dividing a square into two squares).[84] Diophantus also made significant advances in notation, the Arithmetica being the first instance of algebraic symbolism and syncopation.[83] The Hagia Sophia was designed by mathematicians Anthemius of Tralles and Isidore of Miletus. Among the last great Greek mathematicians is Pappus of Alexandria (4th century AD). He is known for his hexagon theorem and centroid theorem, as well as the Pappus configuration and Pappus graph. His Collection is a major source of knowledge on Greek mathematics as most of it has survived.[85] Pappus is considered the last major innovator in Greek mathematics, with subsequent work consisting mostly of commentaries on earlier work. The first woman mathematician recorded by history was Hypatia of Alexandria (AD 350–415). She succeeded her father (Theon of Alexandria) as Librarian at the Great Library[citation needed] and wrote many works on applied mathematics. Because of a political dispute, the Christian community in Alexandria had her stripped publicly and executed.[86] Her death is sometimes taken as the end of the era of the Alexandrian Greek mathematics, although work did continue in Athens for another century with figures such as Proclus, Simplicius and Eutocius.[87] Although Proclus and Simplicius were more philosophers than mathematicians, their commentaries on earlier works are valuable sources on Greek mathematics. The closure of the neo-Platonic Academy of Athens by the emperor Justinian in 529 AD is traditionally held as marking the end of the era of Greek mathematics, although the Greek tradition continued unbroken in the Byzantine empire with mathematicians such as Anthemius of Tralles and Isidore of Miletus, the architects of the Hagia Sophia.[88] Nevertheless, Byzantine mathematics consisted mostly of commentaries, with little in the way of innovation, and the centers of mathematical innovation were to be found elsewhere by this time.[89] Further information: Roman abacus and Roman numerals Equipment used by an ancient Roman land surveyor (gromatici), found at the site of Aquincum, modern Budapest, Hungary Although ethnic Greek mathematicians continued under the rule of the late Roman Republic and subsequent Roman Empire, there were no noteworthy native Latin mathematicians in comparison.[90][91] Ancient Romans such as Cicero (106–43 BC), an influential Roman statesman who studied mathematics in Greece, believed that Roman surveyors and calculators were far more interested in applied mathematics than the theoretical mathematics and geometry that were prized by the Greeks.[92] It is unclear if the Romans first derived their numerical system directly from the Greek precedent or from Etruscan numerals used by the Etruscan civilization centered in what is now Tuscany, central Italy.[93] Using calculation, Romans were adept at both instigating and detecting financial fraud, as well as managing taxes for the treasury.[94] Siculus Flaccus, one of the Roman gromatici (i.e. land surveyor), wrote the Categories of Fields, which aided Roman surveyors in measuring the surface areas of allotted lands and territories.[95] Aside from managing trade and taxes, the Romans also regularly applied mathematics to solve problems in engineering, including the erection of architecture such as bridges, road-building, and preparation for military campaigns.[96] Arts and crafts such as Roman mosaics, inspired by previous Greek designs, created illusionist geometric patterns and rich, detailed scenes that required precise measurements for each tessera tile, the opus tessellatum pieces on average measuring eight millimeters square and the finer opus vermiculatum pieces having an average surface of four millimeters square.[97][98] The creation of the Roman calendar also necessitated basic mathematics. The first calendar allegedly dates back to 8th century BC during the Roman Kingdom and included 356 days plus a leap year every other year.[99] In contrast, the lunar calendar of the Republican era contained 355 days, roughly ten-and-one-fourth days shorter than the solar year, a discrepancy that was solved by adding an extra month into the calendar after the 23rd of February.[100] This calendar was supplanted by the Julian calendar, a solar calendar organized by Julius Caesar (100–44 BC) and devised by Sosigenes of Alexandria to include a leap day every four years in a 365-day cycle.[101] This calendar, which contained an error of 11 minutes and 14 seconds, was later corrected by the Gregorian calendar organized by Pope Gregory XIII (r. 1572–1585), virtually the same solar calendar used in modern times as the international standard calendar.[102] At roughly the same time, the Han Chinese and the Romans both invented the wheeled odometer device for measuring distances traveled, the Roman model first described by the Roman civil engineer and architect Vitruvius (c. 80 BC – c. 15 BC).[103] The device was used at least until the reign of emperor Commodus (r. 177 – 192 AD), but its design seems to have been lost until experiments were made during the 15th century in Western Europe.[104] Perhaps relying on similar gear-work and technology found in the Antikythera mechanism, the odometer of Vitruvius featured chariot wheels measuring 4 feet (1.2 m) in diameter turning four-hundred times in one Roman mile (roughly 4590 ft/1400 m). With each revolution, a pin-and-axle device engaged a 400-tooth cogwheel that turned a second gear responsible for dropping pebbles into a box, each pebble representing one mile traversed.[105] Main article: Chinese mathematics Further information: Book on Numbers and Computation See also: History of science § Chinese mathematics The Tsinghua Bamboo Slips, containing the world's earliest decimal multiplication table, dated 305 BC during the Warring States period An analysis of early Chinese mathematics has demonstrated its unique development compared to other parts of the world, leading scholars to assume an entirely independent development.[106] The oldest extant mathematical text from China is the Zhoubi Suanjing (周髀算經), variously dated to between 1200 BC and 100 BC, though a date of about 300 BC during the Warring States Period appears reasonable.[107] However, the Tsinghua Bamboo Slips, containing the earliest known decimal multiplication table (although ancient Babylonians had ones with a base of 60), is dated around 305 BC and is perhaps the oldest surviving mathematical text of China.[49] Counting rod numerals Of particular note is the use in Chinese mathematics of a decimal positional notation system, the so-called "rod numerals" in which distinct ciphers were used for numbers between 1 and 10, and additional ciphers for powers of ten.[108] Thus, the number 123 would be written using the symbol for "1", followed by the symbol for "100", then the symbol for "2" followed by the symbol for "10", followed by the symbol for "3". This was the most advanced number system in the world at the time, apparently in use several centuries before the common era and well before the development of the Indian numeral system.[109] Rod numerals allowed the representation of numbers as large as desired and allowed calculations to be carried out on the suan pan, or Chinese abacus. The date of the invention of the suan pan is not certain, but the earliest written mention dates from AD 190, in Xu Yue's Supplementary Notes on the Art of Figures. The oldest extant work on geometry in China comes from the philosophical Mohist canon c. 330 BC, compiled by the followers of Mozi (470–390 BC). The Mo Jing described various aspects of many fields associated with physical science, and provided a small number of geometrical theorems as well.[110] It also defined the concepts of circumference, diameter, radius, and volume.[111] The Nine Chapters on the Mathematical Art, one of the earliest surviving mathematical texts from China (2nd century AD). In 212 BC, the Emperor Qin Shi Huang commanded all books in the Qin Empire other than officially sanctioned ones be burned. This decree was not universally obeyed, but as a consequence of this order little is known about ancient Chinese mathematics before this date. After the book burning of 212 BC, the Han dynasty (202 BC–220 AD) produced works of mathematics which presumably expanded on works that are now lost. The most important of these is The Nine Chapters on the Mathematical Art, the full title of which appeared by AD 179, but existed in part under other titles beforehand. It consists of 246 word problems involving agriculture, business, employment of geometry to figure height spans and dimension ratios for Chinese pagoda towers, engineering, surveying, and includes material on right triangles.[107] It created mathematical proof for the Pythagorean theorem,[112] and a mathematical formula for Gaussian elimination.[113] The treatise also provides values of π,[107] which Chinese mathematicians originally approximated as 3 until Liu Xin (d. 23 AD) provided a figure of 3.1457 and subsequently Zhang Heng (78–139) approximated pi as 3.1724,[114] as well as 3.162 by taking the square root of 10.[115][116] Liu Hui commented on the Nine Chapters in the 3rd century AD and gave a value of π accurate to 5 decimal places (i.e. 3.14159).[117][118] Though more of a matter of computational stamina than theoretical insight, in the 5th century AD Zu Chongzhi computed the value of π to seven decimal places (between 3.1415926 and 3.1415927), which remained the most accurate value of π for almost the next 1000 years.[117][119] He also established a method which would later be called Cavalieri's principle to find the volume of a sphere.[120] The high-water mark of Chinese mathematics occurred in the 13th century during the latter half of the Song dynasty (960–1279), with the development of Chinese algebra. The most important text from that period is the Precious Mirror of the Four Elements by Zhu Shijie (1249–1314), dealing with the solution of simultaneous higher order algebraic equations using a method similar to Horner's method.[117] The Precious Mirror also contains a diagram of Pascal's triangle with coefficients of binomial expansions through the eighth power, though both appear in Chinese works as early as 1100.[121] The Chinese also made use of the complex combinatorial diagram known as the magic square and magic circles, described in ancient times and perfected by Yang Hui (AD 1238–1298).[121] Even after European mathematics began to flourish during the Renaissance, European and Chinese mathematics were separate traditions, with significant Chinese mathematical output in decline from the 13th century onwards. Jesuit missionaries such as Matteo Ricci carried mathematical ideas back and forth between the two cultures from the 16th to 18th centuries, though at this point far more mathematical ideas were entering China than leaving.[121] Japanese mathematics, Korean mathematics, and Vietnamese mathematics are traditionally viewed as stemming from Chinese mathematics and belonging to the Confucian-based East Asian cultural sphere.[122] Korean and Japanese mathematics were heavily influenced by the algebraic works produced during China's Song dynasty, whereas Vietnamese mathematics was heavily indebted to popular works of China's Ming dynasty (1368–1644).[123] For instance, although Vietnamese mathematical treatises were written in either Chinese or the native Vietnamese Chữ Nôm script, all of them followed the Chinese format of presenting a collection of problems with algorithms for solving them, followed by numerical answers.[124] Mathematics in Vietnam and Korea were mostly associated with the professional court bureaucracy of mathematicians and astronomers, whereas in Japan it was more prevalent in the realm of private schools.[125] Main article: Indian mathematics Further information: History of science § Indian mathematics See also: History of the Hindu–Arabic numeral system The numerals used in the Bakhshali manuscript, dated between the 2nd century BC and the 2nd century AD. Indian numerals in stone and copper inscriptions[126]Ancient Brahmi numerals in a part of India The earliest civilization on the Indian subcontinent is the Indus Valley civilization (mature second phase: 2600 to 1900 BC) that flourished in the Indus river basin. Their cities were laid out with geometric regularity, but no known mathematical documents survive from this civilization.[127] The oldest extant mathematical records from India are the Sulba Sutras (dated variously between the 8th century BC and the 2nd century AD),[128] appendices to religious texts which give simple rules for constructing altars of various shapes, such as squares, rectangles, parallelograms, and others.[129] As with Egypt, the preoccupation with temple functions points to an origin of mathematics in religious ritual.[128] The Sulba Sutras give methods for constructing a circle with approximately the same area as a given square, which imply several different approximations of the value of π.[130][131][a] In addition, they compute the square root of 2 to several decimal places, list Pythagorean triples, and give a statement of the Pythagorean theorem.[131] All of these results are present in Babylonian mathematics, indicating Mesopotamian influence.[128] It is not known to what extent the Sulba Sutras influenced later Indian mathematicians. As in China, there is a lack of continuity in Indian mathematics; significant advances are separated by long periods of inactivity.[128] Pāṇini (c. 5th century BC) formulated the rules for Sanskrit grammar.[132] His notation was similar to modern mathematical notation, and used metarules, transformations, and recursion.[133] Pingala (roughly 3rd–1st centuries BC) in his treatise of prosody uses a device corresponding to a binary numeral system.[134][135] His discussion of the combinatorics of meters corresponds to an elementary version of the binomial theorem. Pingala's work also contains the basic ideas of Fibonacci numbers (called mātrāmeru).[136] The next significant mathematical documents from India after the Sulba Sutras are the Siddhantas, astronomical treatises from the 4th and 5th centuries AD (Gupta period) showing strong Hellenistic influence.[137] They are significant in that they contain the first instance of trigonometric relations based on the half-chord, as is the case in modern trigonometry, rather than the full chord, as was the case in Ptolemaic trigonometry.[138] Through a series of translation errors, the words "sine" and "cosine" derive from the Sanskrit "jiya" and "kojiya".[138] Explanation of the sine rule in Yuktibhāṣā Around 500 AD, Aryabhata wrote the Aryabhatiya, a slim volume, written in verse, intended to supplement the rules of calculation used in astronomy and mathematical mensuration, though with no feeling for logic or deductive methodology.[139] It is in the Aryabhatiya that the decimal place-value system first appears. Several centuries later, the Muslim mathematician Abu Rayhan Biruni described the Aryabhatiya as a "mix of common pebbles and costly crystals".[140] In the 7th century, Brahmagupta identified the Brahmagupta theorem, Brahmagupta's identity and Brahmagupta's formula, and for the first time, in Brahma-sphuta-siddhanta, he lucidly explained the use of zero as both a placeholder and decimal digit, and explained the Hindu–Arabic numeral system.[141] It was from a translation of this Indian text on mathematics (c. 770) that Islamic mathematicians were introduced to this numeral system, which they adapted as Arabic numerals. Islamic scholars carried knowledge of this number system to Europe by the 12th century, and it has now displaced all older number systems throughout the world. Various symbol sets are used to represent numbers in the Hindu–Arabic numeral system, all of which evolved from the Brahmi numerals. Each of the roughly dozen major scripts of India has its own numeral glyphs. In the 10th century, Halayudha's commentary on Pingala's work contains a study of the Fibonacci sequence[142] and Pascal's triangle,[143] and describes the formation of a matrix.[citation needed] In the 12th century, Bhāskara II,[144] who lived in southern India, wrote extensively on all then known branches of mathematics. His work contains mathematical objects equivalent or approximately equivalent to infinitesimals, the mean value theorem and the derivative of the sine function although he did not develop the notion of a derivative.[145][146] In the 14th century, Narayana Pandita completed his Ganita Kaumudi.[147] Also in the 14th century, Madhava of Sangamagrama, the founder of the Kerala School of Mathematics, found the Madhava–Leibniz series and obtained from it a transformed series, whose first 21 terms he used to compute the value of π as 3.14159265359. Madhava also found the Madhava-Gregory series to determine the arctangent, the Madhava-Newton power series to determine sine and cosine and the Taylor approximation for sine and cosine functions.[148] In the 16th century, Jyesthadeva consolidated many of the Kerala School's developments and theorems in the Yukti-bhāṣā.[149][150] It has been argued that certain ideas of calculus like infinite series and taylor series of some trigonometry functions, were transmitted to Europe in the 16th century[6] via Jesuit missionaries and traders who were active around the ancient port of Muziris at the time and, as a result, directly influenced later European developments in analysis and calculus.[151] However, other scholars argue that the Kerala School did not formulate a systematic theory of differentiation and integration, and that there is not any direct evidence of their results being transmitted outside Kerala.[152][153][154][155] Main article: Mathematics in medieval Islam See also: History of the Hindu–Arabic numeral system Page from The Compendious Book on Calculation by Completion and Balancing by Muhammad ibn Mūsā al-Khwārizmī (c. AD 820) The Islamic Empire established across the Middle East, Central Asia, North Africa, Iberia, and in parts of India in the 8th century made significant contributions towards mathematics. Although most Islamic texts on mathematics were written in Arabic, they were not all written by Arabs, since much like the status of Greek in the Hellenistic world, Arabic was used as the written language of non-Arab scholars throughout the Islamic world at the time.[156] In the 9th century, the Persian mathematician Muḥammad ibn Mūsā al-Khwārizmī wrote an important book on the Hindu–Arabic numerals and one on methods for solving equations. His book On the Calculation with Hindu Numerals,

Hungarian mathematician János Bolyai, independently defined and studied hyperbolic geometry, where uniqueness of parallels no longer holds. In this geometry the sum of angles in a triangle add up to less than 180°. Elliptic geometry was developed later in the 19th century by the German mathematician Bernhard Riemann; here no parallel can be found and the angles in a triangle add up to more than 180°. Riemann also developed Riemannian geometry, which unifies and vastly generalizes the three types of geometry, and he defined the concept of a manifold, which generalizes the ideas of curves and surfaces, and set the mathematical foundations for the theory of general relativity.[196] The 19th century saw the beginning of a great deal of abstract algebra. Hermann Grassmann in Germany gave a first version of vector spaces, William Rowan Hamilton in Ireland developed noncommutative algebra.[citation needed] The British mathematician George Boole devised an algebra that soon evolved into what is now called Boolean algebra, in which the only numbers were 0 and 1. Boolean algebra is the starting point of mathematical logic and has important applications in electrical engineering and computer science.[citation needed][197] Augustin-Louis Cauchy, Bernhard Riemann, and Karl Weierstrass reformulated the calculus in a more rigorous fashion.[citation needed] Also, for the first time, the limits of mathematics were explored. Niels Henrik Abel, a Norwegian, and Évariste Galois, a Frenchman, proved that there is no general algebraic method for solving polynomial equations of degree greater than four (Abel–Ruffini theorem).[198] Other 19th-century mathematicians used this in their proofs that straight edge and compass alone are not sufficient to trisect an arbitrary angle, to construct the side of a cube twice the volume of a given cube, nor to construct a square equal in area to a given circle.[citation needed] Mathematicians had vainly attempted to solve all of these problems since the time of the ancient Greeks.[citation needed] On the other hand, the limitation of three dimensions in geometry was surpassed in the 19th century through considerations of parameter space and hypercomplex numbers.[citation needed] Abel and Galois's investigations into the solutions of various polynomial equations laid the groundwork for further developments of group theory, and the associated fields of abstract algebra. In the 20th century physicists and other scientists have seen group theory as the ideal way to study symmetry.[citation needed] Georg Cantor In the later 19th century, Georg Cantor established the first foundations of set theory, which enabled the rigorous treatment of the notion of infinity and has become the common language of nearly all mathematics. Cantor's set theory, and the rise of mathematical logic in the hands of Peano, L.E.J. Brouwer, David Hilbert, Bertrand Russell, and A.N. Whitehead, initiated a long running debate on the foundations of mathematics.[citation needed] The 19th century saw the founding of a number of national mathematical societies: the London Mathematical Society in 1865,[199] the Société Mathématique de France in 1872,[200] the Circolo Matematico di Palermo in 1884,[201][202] the Edinburgh Mathematical Society in 1883,[203] and the American Mathematical Society in 1888.[204] The first international, special-interest society, the Quaternion Society, was formed in 1899, in the context of a vector controversy.[205] In 1897, Kurt Hensel introduced p-adic numbers.[206] The 20th century saw mathematics become a major profession. By the end of the century, thousands of new Ph.D.s in mathematics were being awarded every year, and jobs were available in both teaching and industry.[207] An effort to catalogue the areas and applications of mathematics was undertaken in Klein's encyclopedia.[208] In a 1900 speech to the International Congress of Mathematicians, David Hilbert set out a list of 23 unsolved problems in mathematics.[209] These problems, spanning many areas of mathematics, formed a central focus for much of 20th-century mathematics. Today, 10 have been solved, 7 are partially solved, and 2 are still open. The remaining 4 are too loosely formulated to be stated as solved or not.[210] A map illustrating the Four Color Theorem Notable historical conjectures were finally proven. In 1976, Wolfgang Haken and Kenneth Appel proved the four color theorem, controversial at the time for the use of a computer to do so.[211] Andrew Wiles, building on the work of others, proved Fermat's Last Theorem in 1995.[212] Paul Cohen and Kurt Gödel proved that the continuum hypothesis is independent of (could neither be proved nor disproved from) the standard axioms of set theory.[213] In 1998, Thomas Callister Hales proved the Kepler conjecture, also using a computer.[214] Mathematical collaborations of unprecedented size and scope took place. An example is the classification of finite simple groups (also called the "enormous theorem"), whose proof between 1955 and 2004 required 500-odd journal articles by about 100 authors, and filling tens of thousands of pages.[215] A group of French mathematicians, including Jean Dieudonné and André Weil, publishing under the pseudonym "Nicolas Bourbaki", attempted to exposit all of known mathematics as a coherent rigorous whole. The resulting several dozen volumes has had a controversial influence on mathematical education.[216] Newtonian (red) vs. Einsteinian orbit (blue) of a lone planet orbiting a star, with relativistic precession of apsides Differential geometry came into its own when Albert Einstein used it in general relativity.[citation needed] Entirely new areas of mathematics such as mathematical logic, topology, and John von Neumann's game theory changed the kinds of questions that could be answered by mathematical methods.[citation needed] All kinds of structures were abstracted using axioms and given names like metric spaces, topological spaces etc.[citation needed] As mathematicians do, the concept of an abstract structure was itself abstracted and led to category theory.[citation needed] Grothendieck and Serre recast algebraic geometry using sheaf theory.[citation needed] Large advances were made in the qualitative study of dynamical systems that Poincaré had begun in the 1890s.[citation needed] Measure theory was developed in the late 19th and early 20th centuries. Applications of measures include the Lebesgue integral, Kolmogorov's axiomatisation of probability theory, and ergodic theory.[citation needed] Knot theory greatly expanded.[citation needed] Quantum mechanics led to the development of functional analysis,[citation needed] a branch of mathematics that was greatly developed by Stefan Banach and his collaborators who formed the Lwów School of Mathematics.[217] Other new areas include Laurent Schwartz's distribution theory, fixed point theory, singularity theory and René Thom's catastrophe theory, model theory, and Mandelbrot's fractals.[citation needed] Lie theory with its Lie groups and Lie algebras became one of the major areas of study.[218] Non-standard analysis, introduced by Abraham Robinson, rehabilitated the infinitesimal approach to calculus, which had fallen into disrepute in favour of the theory of limits, by extending the field of real numbers to the Hyperreal numbers which include infinitesimal and infinite quantities.[citation needed] An even larger number system, the surreal numbers were discovered by John Horton Conway in connection with combinatorial games.[citation needed] The development and continual improvement of computers, at first mechanical analog machines and then digital electronic machines, allowed industry to deal with larger and larger amounts of data to facilitate mass production and distribution and communication, and new areas of mathematics were developed to deal with this: Alan Turing's computability theory; complexity theory; Derrick Henry Lehmer's use of ENIAC to further number theory and the Lucas–Lehmer primality test; Rózsa Péter's recursive function theory; Claude Shannon's information theory; signal processing; data analysis; optimization and other areas of operations research.[citation needed] In the preceding centuries much mathematical focus was on calculus and continuous functions, but the rise of computing and communication networks led to an increasing importance of discrete concepts and the expansion of combinatorics including graph theory. The speed and data processing abilities of computers also enabled the handling of mathematical problems that were too time-consuming to deal with by pencil and paper calculations, leading to areas such as numerical analysis and symbolic computation.[citation needed] Some of the most important methods and algorithms of the 20th century are: the simplex algorithm, the fast Fourier transform, error-correcting codes, the Kalman filter from control theory and the RSA algorithm of public-key cryptography.[citation needed] At the same time, deep insights were made about the limitations to mathematics. In 1929 and 1930, it was proved[by whom?] the truth or falsity of all statements formulated about the natural numbers plus either addition or multiplication (but not both), was decidable, i.e. could be determined by some algorithm.[citation needed] In 1931, Kurt Gödel found that this was not the case for the natural numbers plus both addition and multiplication; this system, known as Peano arithmetic, was in fact incomplete. (Peano arithmetic is adequate for a good deal of number theory, including the notion of prime number.) A consequence of Gödel's two incompleteness theorems is that in any mathematical system that includes Peano arithmetic (including all of analysis and geometry), truth necessarily outruns proof, i.e. there are true statements that cannot be proved within the system. Hence mathematics cannot be reduced to mathematical logic, and David Hilbert's dream of making all of mathematics complete and consistent needed to be reformulated.[citation needed] The absolute value of the Gamma function on the complex plane One of the more colorful figures in 20th-century mathematics was Srinivasa Aiyangar Ramanujan (1887–1920), an Indian autodidact[219] who conjectured or proved over 3000 theorems[citation needed], including properties of highly composite numbers,[220] the partition function[219] and its asymptotics,[221] and mock theta functions.[219] He also made major investigations in the areas of gamma functions,[222][223] modular forms,[219] divergent series,[219] hypergeometric series[219] and prime number theory.[219] Paul Erdős published more papers than any other mathematician in history,[224] working with hundreds of collaborators. Mathematicians have a game equivalent to the Kevin Bacon Game, which leads to the Erdős number of a mathematician. This describes the "collaborative distance" between a person and Erdős, as measured by joint authorship of mathematical papers.[225][226] Emmy Noether has been described by many as the most important woman in the history of mathematics.[227] She studied the theories of rings, fields, and algebras.[228] As in most areas of study, the explosion of knowledge in the scientific age has led to specialization: by the end of the century, there were hundreds of specialized areas in mathematics, and the Mathematics Subject Classification was dozens of pages long.[229] More and more mathematical journals were published and, by the end of the century, the development of the World Wide Web led to online publishing.[citation needed] See also: List of unsolved problems in mathematics § Problems solved since 1995 In 2000, the Clay Mathematics Institute announced the seven Millennium Prize Problems.[230] In 2003 the Poincaré conjecture was solved by Grigori Perelman (who declined to accept an award, as he was critical of the mathematics establishment).[231] Most mathematical journals now have online versions as well as print versions, and many online-only journals are launched.[232][233] There is an increasing drive toward open access publishing, first made popular by arXiv.[citation needed] Main article: Future of mathematics There are many observable trends in mathematics, the most notable being that the subject is growing ever larger as computers are ever more important and powerful; the volume of data being produced by science and industry, facilitated by computers, continues expanding exponentially. As a result, there is a corresponding growth in the demand for mathematics to help process and understand this big data.[234] Math science careers are also expected to continue to grow, with the US Bureau of Labor Statistics estimating (in 2018) that "employment of mathematical science occupations is projected to grow 27.9 percent from 2016 to 2026."[235] Mathematics portal Archives of American Mathematics Ethnomathematics History of algebra History of arithmetic History of calculus History of combinatorics History of the function concept History of geometry History of group theory History of logic History of mathematicians History of mathematical notation History of measurement History of numbers History of ancient numeral systems Prehistoric counting List of books on history of number systems History of number theory History of statistics History of trigonometry History of writing numbers Kenneth O. May Prize List of important publications in mathematics Lists of mathematicians List of mathematics history topics Mathematical folklore Timeline of mathematics ^ The approximate values for π are 4 x (13/15)2 (3.0044...), 25/8 (3.125), 900/289 (3.11418685...), 1156/361 (3.202216...), and 339/108 (3.1389) ^ a b (Boyer 1991, "Euclid of Alexandria" p. 119) ^ Friberg, J. (1981). "Methods and traditions of Babylonian mathematics. Plimpton 322, Pythagorean triples, and the Babylonian triangle parameter equations", Historia Mathematica, 8, pp. 277–318. ^ Neugebauer, Otto (1969) [1957]. The Exact Sciences in Antiquity. Acta Historica Scientiarum Naturalium et Medicinalium. Vol. 9 (2 ed.). Dover Publications. pp. 1–191. ISBN 978-0-486-22332-2. PMID 14884919. Chap. IV "Egyptian Mathematics and Astronomy", pp. 71–96. ^ Turnbull (1931). "A Manual of Greek Mathematics". Nature. 128 (3235): 5. Bibcode:1931Natur.128..739T. doi:10.1038/128739a0. S2CID 3994109. ^ Heath, Thomas L. (1963). 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When this was first described in English by Charles Whish, in the 1830s, it was heralded as the Indians' discovery of the calculus. This claim and Mādhava's achievements were ignored by Western historians, presumably at first because they could not admit that an Indian discovered the calculus, but later because no one read anymore the Transactions of the Royal Asiatic Society, in which Whish's article was published. The matter resurfaced in the 1950s, and now we have the Sanskrit texts properly edited, and we understand the clever way that Mādhava derived the series without the calculus; but many historians still find it impossible to conceive of the problem and its solution in terms of anything other than the calculus and proclaim that the calculus is what Mādhava found. In this case the elegance and brilliance of Mādhava's mathematics are being distorted as they are buried under the current mathematical solution to a problem to which he discovered an alternate and powerful solution. ^ Bressoud, David (2002). "Was Calculus Invented in India?". College Mathematics Journal. 33 (1): 2–13. doi:10.2307/1558972. JSTOR 1558972. ^ Plofker, Kim (November 2001). "The 'Error' in the Indian "Taylor Series Approximation" to the Sine". Historia Mathematica. 28 (4): 293. doi:10.1006/hmat.2001.2331. It is not unusual to encounter in discussions of Indian mathematics such assertions as that 'the concept of differentiation was understood [in India] from the time of Manjula (... in the 10th century)' [Joseph 1991, 300], or that 'we may consider Madhava to have been the founder of mathematical analysis' (Joseph 1991, 293), or that Bhaskara II may claim to be 'the precursor of Newton and Leibniz in the discovery of the principle of the differential calculus' (Bag 1979, 294).... The points of resemblance, particularly between early European calculus and the Keralese work on power series, have even inspired suggestions of a possible transmission of mathematical ideas from the Malabar coast in or after the 15th century to the Latin scholarly world (e.g., in (Bag 1979, 285))... It should be borne in mind, however, that such an emphasis on the similarity of Sanskrit (or Malayalam) and Latin mathematics risks diminishing our ability fully to see and comprehend the former. To speak of the Indian 'discovery of the principle of the differential calculus' somewhat obscures the fact that Indian techniques for expressing changes in the Sine by means of the Cosine or vice versa, as in the examples we have seen, remained within that specific trigonometric context. The differential 'principle' was not generalized to arbitrary functions – in fact, the explicit notion of an arbitrary function, not to mention that of its derivative or an algorithm for taking the derivative, is irrelevant here ^ Katz, Victor J. (June 1995). "Ideas of Calculus in Islam and India" (PDF). Mathematics Magazine. 68 (3): 163–74. doi:10.2307/2691411. JSTOR 2691411. ^ Abdel Haleem, Muhammad A. S. "The Semitic Languages", "Arabic became the language of scholarship in science and philosophy in the 9th century when the 'translation movement' saw concerted work on translations of Greek, Indian, Persian and Chinese, medical, philosophical and scientific texts", p. 811. ^ (Boyer 1991, "The Arabic Hegemony" p. 230) "The six cases of equations given above exhaust all possibilities for linear and quadratic equations having positive root. So systematic and exhaustive was al-Khwārizmī's exposition that his readers must have had little difficulty in mastering the solutions." ^ Gandz and Saloman (1936). "The sources of Khwarizmi's algebra", Osiris i, pp. 263–77: "In a sense, Khwarizmi is more entitled to be called "the father of algebra" than Diophantus because Khwarizmi is the first to teach algebra in an elementary form and for its own sake, Diophantus is primarily concerned with the theory of numbers". ^ (Boyer 1991, "The Arabic Hegemony" p. 229) "It is not certain just what the terms al-jabr and muqabalah mean, but the usual interpretation is similar to that implied in the translation above. The word al-jabr presumably meant something like "restoration" or "completion" and seems to refer to the transposition of subtracted terms to the other side of an equation; the word muqabalah is said to refer to "reduction" or "balancing" – that is, the cancellation of like terms on opposite sides of the equation." ^ Rashed, R.; Armstrong, Angela (1994). The Development of Arabic Mathematics. Springer. pp. 11–12. ISBN 978-0-7923-2565-9. OCLC 29181926. ^ Sesiano, Jacques (1997). "Abū Kāmil". 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